

Usermanual

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OCPBASIC

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1 LICENCE

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2 Installing IPbasic and OCPbasic

2.1 Fast Installing Procedure

If you do not like to build IPBASIC and OCPBASIC yourself you can get precompiled packages for Ubuntu 22.04.

2.1.1 Installing via apt-get on Ubuntu 22.04

This section will install IPBASIC, OCPBASIC and LAPACK_WRAPPER using an aptrepository and the linear algebra packages libblas-dev and liblapack-dev. First download and activate the ocpbasic gpg key and install it to your system.

Comment 2.1 Only add software repositories from sources that you trust! Third-party software repositories are not checked for security or reliability and may contain software which is harmful to your computer. The use of these repositorys is at your own risk!

```
$ wget https://archive.ocpbasic.com/dists/jammy/ocpbasic.gpg
$ sudo install -o root -g root -m 644 ocpbasic.gpg \
/etc/apt/trusted.gpg.d/
$ sudo rm ocpbasic.gpg
```

After that you can activate the ocpbasic apt-repository with the command:

```
$ sudo add-apt-repository \
'debu[arch=amd64]uhttp://archive.ocpbasic.com/dists/jammyu/'
```

If the apt-repository is installed correctly you can install OCPBASIC with the command:

\$ sudo apt-get install ocpbasic

The software packages libblas-dev, liblapack-dev, LAPACK_WRAPPER and IPBASIC will then also be installed by apt.

Supported Ubuntu Versions:

Ubuntu version:	apt-repository	
Ubuntu 22.04	'deb [arch=amd64] http://archive.ocpbasic.com/dists/jammy /'	
Ubuntu 20.04	'deb [arch=amd64] http://archive.ocpbasic.com/dists/focal /'	

2 INSTALLING IPBASIC AND OCPBASIC

Ubuntu 18.04	'deb [arch=amd64] http://archive.ocpbasic.com/dists/bionic /'

Table 1:	Supported	Ubuntu	Versions
----------	-----------	--------	----------

Comment 2.2 The maintainer may remove or delete these apt-repositorys any time!

2.2 Selection of linear algebra software packages

Package:	BLAS	LAPACK
BLAS / LAPACK:	~	\checkmark
OpenBLAS	 ✓ 	\checkmark
Intel Math Kernel Library (MKL)	 ✓ 	\checkmark
ATLAS	\sim	\sim
Accelerate - Apple Developer	~	\sim

Table 2: Dense linear algebra software packages used by LAPACK_WRAPPER

Table 2 shows all tested and supported dense linear algebra software packages from LA-PACK_WRAPPER, that can be used by IPBASIC and OCPBASIC.

- ✓ Supported by LAPACK_WRAPPER and tested with IPBASIC and OCPBASIC.
- $\sim\,$ Supported by LAPACK_WRAPPER but not tested with IPBASIC and OCPBASIC.

2.2.1 Selection of linear algebra software packages using qmake

Comment 2.3 The user is responsible for linking against any thirdparty software and checking the licence compatibility.

When using the default build method qmake for building LAPACK_WRAPPER, IPBASIC and OCPBASIC you will find a config.pri file in the src/ folder. In this file you will find a section SET DEFINES, where used packages can be selected. Fild the following lines:

```
#
# Define used linear Algebra.
#
# Only supported by lapack_wrapper:
```

```
# LAPACK_WRAPPER_USE_ACCELERATE, LAPACK_WRAPPER_USE_ATLAS,
#
# Currently supported values by IPbasic and OCPbasic:
# LAPACK_WRAPPER_USE_LAPACK, LAPACK_WRAPPER_USE_OPENBLAS,
# LAPACK_WRAPPER_USE_MKL
#
DEFINES += LAPACK_WRAPPER_USE_LAPACK
```

You can change LAPACK_WRAPPER_USE_LAPACK into your prefered linear algebra software package define.

Comment 2.4 You must only use one single linear algebra software package define!

After selecting your linear algebra software package you have to set the include and shared library path and the linked linear algebra package. Terefore you will find a section SET PATH in the config.pri file.

```
#
#
# Define lib and include path for blas and lapack:
#
BLAS_INCLUDE_PATH = /usr/include/x86_64-linux-gnu/
BLAS_PATH_LIBS = -L/usr/lib/x86_64-linux-gnu/ -lblas
LAPACK_INCLUDE_PATH = /usr/include/x86_64-linux-gnu/
LAPACK_PATH_LIBS = -L/usr/lib/x86_64-linux-gnu/ -llapack
#
# Define lib and include path for Openblas:
#
OPENBLAS_INCLUDE_PATH = \
    /usr/include/x86_64-linux-gnu/openblas-pthread/
OPENBLAS_PATH_LIBS = -L/usr/lib/x86_64-linux-gnu/ \
    _lopenblas
#
# Define lib and include path for intel mkl:
#
```

```
MKL_INCLUDE_PATH = /usr/include/mkl/
MKL_PATH_LIBS = -L/usr/lib/x86_64-linux-gnu/ \
        -lmkl_intel_lp64 \
        -lmkl_intel_thread \
        -lmkl_core \
        -lmkl_rt \
        -liomp5
```

Comment 2.5 Do not delete or comment any unused include and shared library path. When not selected as define the path variables for other linear algebra software packages are ignored anyway.

If you need to link additional software packages and libraries you can activate the define OCPBASIC_ADD_LINKED_LIBS by uncommenting the following line in the section SET DEFINES.

```
#
# Define use of additional linked libs.
#
DEFINES += OCPBASIC_ADD_LINKED_LIBS
```

After activating the define OCPBASIC_ADD_LINKED_LIBS you can link additional libraries in the variable ADD_LINKED_LIBS in section SET PATH of the config.pri file.

```
#
# Additional linked libs:
#
ADD_LINKED_LIBS = -lgfortran
```

2.2.2 Selection of linear algebra software packages using other buildsystem

Comment 2.6 The user is responsible for linking against any thirdparty software and checking the licence compatibility.

The linear algebra software packages can be selected in the configuration file of LA-PACK_WRAPPER lapack_wrapper_config.hh. Uncomment the linear algebra software package define.

```
/*\
```

```
! values
```

```
! LAPACK_WRAPPER_USE_ACCELERATE, LAPACK_WRAPPER_USE_ATLAS,
```

! LAPACK_WRAPPER_USE_OPENBLAS, LAPACK_WRAPPER_USE_LAPACK, ! LAPACK_WRAPPER_USE_MKL */

#define LAPACK_WRAPPER_USE_LAPACK 1

Make sure you have your the corresponding linear algebra software package installed and linked with your buildsystem.

2.3 Selection supported of linear algebra solvers

Comment 2.7 By default a LAPACK banded KKT solver can be used with OCPBASIC. When only using OCPBASIC with its LAPACK banded KKT solver you do not need to install any other linear solver, which is not included in LAPACK.

Package:	IPBASIC	OCPBASIC
HSL	~	\checkmark
LAPACK banded KKT	x	\checkmark
Linear solver interface	\checkmark	x

Table 3: Linear algebra solver packages used by IPBASICand OCPBASIC

IPBASIC provides a flexible linear solver interface, where users can define own linear solvers, which are then used by IPBASIC. OCPBASIC provides a powerfull structure exploiting linear solver for KKT systems using LAPACK.

For speedtesting also an interface for the sparse linear solvers of HSL is included in IPBASIC. If activated in IPBASIC, MA57 from HSL can also be used in OCPBASIC. Since IPBASIC and OCPBASIC do not provide any copy of HSL, by default the support of HSL and MA57 is not active. If you do have a copy of HSL and you are using qmake, you can activate the support in the SET DEFINES section of the file config.pri by uncommenting the IPBASIC_USE_HSL define.

Comment 2.8 The user is responsible for linking against any thirdparty software and checking the licence compatibility.

```
#
# Define use of hsl MA57 and MA48. Make sure a libhsl.so
# with all methods of MA57 and MA48 is installed to your
# system.
#
```

#DEFINES += IPBASIC_USE_HSL

Do not forget to also set a path to the hsl.so library. This can by done in the SET PATH section of the file config.pri.

```
#
# Define lib path for hsl:
#
HSL_LIB_PATH = /usr/lib/
```

If you do have a copy of HSL and you are using an other buildsystem, you can activate the support in the file IPbasic_Config.h.

```
// Flag for use of hsl:
#define IPBASIC_USE_HSL
```

2.4 Building and Installing IPbasic and OCPbasic on Ubuntu 20.04

2.4.1 Prepare buildsystem on Ubuntu 20.04

As default qmake is used for building IPBASIC and OCPBASIC. The installing procedure was tested on a fresh Ubuntu 20.04 LTS installation. The buildsystem can by installed by using the shell commands:

\$ sudo apt-get install g++ -y
\$ sudo apt-get install build-essential -y
\$ sudo apt-get install qt5-default -y

Comment 2.9 The user is responsible for installing and linking against any thirdparty software and checking the licence compatibility.

2.4.2 Install preferred linear algebra package

Lapack and Blas

Installing lapack and blas by using the shell command:

\$ sudo apt-get install libblas-dev liblapack-dev -y

OpenBLAS

Installing OpenBLAS by using the shell command:

\$ sudo apt-get install libopenblas-dev -y

Intel MKL

Installing Intel MKL by using the shell command:

\$ sudo apt-get install intel-mkl-full -y

Comment 2.10 The user is responsible for installing and linking against any thirdparty software and checking the licence compatibility.

Comment 2.11 After installing the linear algebra packages you have to change to the correct linear algebra package define and enter the correct path variables in the config.pri file as described in section 2.2.1.

2.4.3 Build and install lapack_wrapper, IPbasic and OCPbasic

Open a terminal in the OCPbasicMaster/ folder. First create a build folder and change your directory into the build folder.

\$ mkdir build
\$ cd build

Use qmake to generate a Makefile from the OCPbasicMaster/ folder.

```
$ qmake ../
```

Build LAPACK_WRAPPER, IPBASIC and OCPBASIC using make.

```
$ make
```

Install LAPACK_WRAPPER, IPBASIC and OCPBASIC system wide using make.

```
$ sudo make install
```

After installing LAPACK_WRAPPER, IPBASIC and OCPBASIC you can check the installation using make.

\$ make check

Comment 2.12 The command make check will execute some unittests. Some unittests return errors, if HSL is not used. Anyway by default OCPBASIC is compiled without HSL and you can use the LAPACK banded KKT solver instead.

3 Using OCPbasic

OCPBASIC is an optimizer for solving optimal control problems. OCPBASIC is using a direct discretization method for state-constrained optimal control problems and an interior-point method for solving the corresponding nonlinear optimazation problem. The optimizer uses structure exploitation in the fully discretized, state constrained optimal control problem for solving the linear equation systems of the interior-point method.

3.1 Optimal Control Problems in OCPbasic

The class of optimal control problems, which can be solved by OCPBASIC is defined by the following OCP.

Problem 3.1 (Optimal Control Problem (OCP))

Find the states $y : [t_0, t_f] \to \mathbb{R}^{n_y}$, the parameter vector $p \in \mathbb{R}^{n_p}$ and the controls $u : [t_0, t_f] \to \mathbb{R}^{n_u}$, such that

$$\varphi(y(t_0), y(t_f), p)$$

is minimal, subject to the nonlinear constraints

$$\begin{aligned} y'(t) &= F(t, y(t), u(t), p) \\ \psi_{\min} &\leq \psi(y(t_0), y(t_f), p) \leq \psi_{\max} \\ v_{\min} &\leq v(t, y(t), u(t), p) \leq v_{\max} \end{aligned} \qquad t \in [t_0, t_f] \end{aligned}$$

and the box conditions

$$y_{\min} \le y(t) \le y_{\max} \qquad t \in [t_0, t_f]$$
$$u_{\min} \le u(t) \le u_{\max} \qquad t \in [t_0, t_f]$$
$$p_{\min} \le p \le p_{\max}.$$

Herein, the functions

$$\varphi : \mathbb{R}^{n_y} \times \mathbb{R}^{n_y} \times \mathbb{R}^{n_p} \to \mathbb{R}$$

$$F : [t_0, t_f] \times \mathbb{R}^{n_y} \times \mathbb{R}^{n_u} \times \mathbb{R}^{n_p} \to \mathbb{R}^{n_y}$$

$$\psi : \mathbb{R}^{n_y} \times \mathbb{R}^{n_y} \times \mathbb{R}^{n_p} \to \mathbb{R}^{n_\psi}$$

$$v : [t_0, t_f] \times \mathbb{R}^{n_y} \times \mathbb{R}^{n_u} \times \mathbb{R}^{n_p} \to \mathbb{R}^{n_v}.$$

are user-defined functions.

The definition of the interior-point-method used by IPBASIC and more information about OCPBASIC can be found in [1] and [2].

3.2 Parameter settings of OCPbasic

3.2.1 Included parameter settings from IPbasic

 μ_0 :

Notation:	mu_0
Description:	Startvalue of the barrier parameter μ .
Value range:	[0, 1]
Default value:	0.5

 ϵ_{tol} :

Notation:	epsilon_tol
Description:	Tolerance of the interior-point-method.
Value range:	[0, 1]
Default value:	1e - 8

 Θ^{μ} :

Notation:	Theta_mu
Description:	Correction parameter of the barrier parameter μ .
Value range:	(1,2)
Default value:	$\left \begin{array}{c} \frac{2}{3} \end{array} \right $

 κ_ϵ :

Notation:	kappa_epsilon
Description:	Optimazation constant.
Value range:	> 0
Default value:	10.0

 κ_{μ} :

Notation:	kappa_mu
Description:	Optimazation constant.
Value range:	(0,1)
Default value:	0.9

 \max_{iter} :

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Notation:	max_iter
Description:	Max number of iterations.
Value range:	> 0
Default value:	1000

 α_{\max} :

Notation:	alpha_max
Description:	Max stepsize.
Value range:	> 0
Default value:	1

 α_{tol} :

Notation:	tol_alpha
Description:	Min stepsize.
Value range:	> 0
Default value:	2e - 8

 $\sigma \textbf{:}$

Notation:	sigma
Description:	Parameter for the armijo-rule.
Value range:	(0,1)
Default value:	0.01

 β :

Notation:	beta
Description:	Parameter for the armijo-rule.
Value range:	(0, 1)
Default value:	0.9

 au_{\min} :

Notation:	tau_min
Description:	Distance parameter for the bounds.
Value range:	(0,1)
Default value:	1e-4

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Printlevel:

Notation:	PrintLev
Description:	Printlevel of IPbasic.
Value range:	PRINT_NONE = No printing.
	PRINT_LESS = Printing only solution.
	PRINT_MORE = Printing iterations and solution.
	$\mathbf{PRINT}_{\mathbf{MARKED}} = \mathbf{Printing}$ iterations and solution with colors.
Default value:	PRINT_MORE

 $\lambda_{ ext{tol}}$:

Notation:	lambda_tol
Description:	Tolerance of the lagrange multipliers.
Value range:	(0,1)
Default value:	1e - 10

RegDiagMatrixEqu:

Notation:	RegDiagMatrixEqu
Description:	Regularisation parameter of the diagonal matrix of the equations.
Value range:	
Default value:	0

${\bf RegDiagMatrixInitialGuessLambda:}$

Notation:	RegDiagMatrixInitialGuessLambda
Description:	Regularisation parameter of the diagonal matrix lagrange multipliers.
Value range:	
Default value:	1e-9

$CheckMatricesFor_INF_NAN:$

Notation:	CheckMatricesFor_INF_NAN
Description:	Tells if matrices are checked due to inf and nan.
Value range:	bool
Default value:	true

LineSearchCouples:

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Notation:	MyLineSearchCouples
Description:	Used LineSearchCouples.
Value range:	$\mathbf{COUPLES}_\mathbf{NONE} = $ No couples.
	$\mathbf{COUPLE_STATE_SLACK} =$
	Couple linesearch of slack and state variables.
	$COUPLE_STATE_SLACK_LAMBDA =$
	Couple linesearch of slack variables, state variables
	and lagrange multipliers.
Default value:	COUPLE_STATE_SLACK

BreakpointMaxTime:

Notation:	BreakpointMaxTime
Description:	Tells, if there is a breakpoint with a maximal time.
Value range:	bool
Default value:	false

maximalTime:

Notation:	maximalTime
Description:	If BreakpointMaxTime, maximal time for IPbasic to solve. [in s]
Value range:	> 0
Default value:	1

3.2.2 New parameter settings in OCPbasic

Regularization:

Notation:	MyRegularization
Description:	Used regularization method.
Value range:	$\mathbf{REGULARIZE}_NONE = No regularization.$
	$\mathbf{REGULARIZE_PARTIAL_EIGENS} =$
	Regularization with partial eigenvalues of the hessian.
	$\mathbf{REGULARIZE}_{\mathbf{GERSCHGORIN}} =$
	Regularization with gerschgorin circles of the hessian.
Default value:	REGULARIZE_NONE

3.2.3 Optimizer flags for OCPbasic

LinearSolverFlag:

Notation:	MyLinearSolverFlag
Description:	Used linear solver method.
Value range:	LINSOL_LAPACK_BANDED_KKT =
	Use of LAPACK banded KKT solver.
	$LINSOL_HSL_MA57 =$
	Use of MA57.
Default value:	LINSOL_LAPACK_BANDED_KKT

LineSearchMethod:

Notation:	UsedLineSearchMethod
Description:	Used linesearch method in IPBASIC.
Value range:	$LINESEARCH_NONE =$
	No linesearch used.
	LINESEARCH_ARMIJO_L1_PENALTY =
	Armijo lines earch with ℓ_1 -penalty-function.
Default value:	LINESEARCH_ARMIJO_L1_PENALTY

4 Examples

4.1 Minimum Energy Problem with OCPbasic

Problem 4.1 (Minimum Energy Problem)

Minimize

 $x_3(1)$

subject to the constraints

$$\begin{aligned} x_1'(t) &= x_2(t) \\ x_2'(t) &= u(t) \\ x_3'(t) &= \frac{1}{2}u(t)^2 \\ x_1(t) &\leq \frac{1}{9} \end{aligned}$$

and with the boundary conditions

$$x_1(0) = 0$$
 $x_1(1) = 0$
 $x_2(0) = 1$ $x_2(1) = -1$
 $x_3(0) = 0.$

One example, which can already be found in [1] and [2] is the Minimum Energy Problem

form Problem 4.1. The numerical solution of Problem is visualized in [1].

In order to implement this OCP with OCPBASIC we need first and second partial derivatives of Problem 4.1. The MinimumEnergyProblem class is defined as a child class of OCPbasic::Problem<double>.

The ode constrains are defined in the suboutine ODE(...). The nonlinear constaints are defined in the suboutine NLCSTR(...). The Boundary constaints are defined in the suboutine BOUND(...) and the target or objective function is defined by the suboutine VARPHI(...).

All first partial derivatives must be defined by suboutines with the nameing format D_\$FUNCTION\$_\$VARIABLE\$.

All second partial derivatives for the constraints are defined by the derived function towards the variables, multiplied by the lagrangian multipliers with the nameing format L_DD_\$FUNCTION\$_\$VARIABLE1\$_\$VARIABLE2\$.

The second partial derivatives for the target or objective function target function towards the variables are not multiplied by the lagrangian multipliers. They have the nameing format L_DD_VARPHI_\$VARIABLE1\$_\$VARIABLE2\$.

Here is a overview for the class header. The full class definition can be found in the examples folder of the source code of OCPBASIC.

```
class MinimumEnergyProblem :
        public OCPbasic::Problem<double>
{
        public:
        /**
        * \brief MinimumEnergyProblem:
                  Constructor of the class
                  MinimumEnergyProblem.
        *
        */
        explicit MinimumEnergyProblem();
        /**
        * \brief ~MinimumEnergyProblem:
                  Virtual destructor of the class
                 MinimumEnergyProblem.
        */
        virtual ~MinimumEnergyProblem();
```

```
/**
* \brief setRangeBoxBounds:
        Initialize the bounds of the constraints.
*
*/
void setRangeBoxBounds();
/**
* \brief ODE:
        Righthand side of the ODE.
*
* \param[in] t
                    Timepoint f$t f$.
* \param[in] y
                    States at f^y(t) f^s.
* \param[in] u
                   Controls at f(t) .
* \param[in] p
                   Parameters f^{0}.
* \param[out] f
                    Righthand side of the ODE as a
                    vector.
*
*/
virtual void ODE(...) const;
/**
* \brief NLCSTR:
        Nonlinear constaints.
*
* \param[in] t
                    Timepoint f$t\f$.
* \param[in] y
                    States at f(t)f.
* \param[in] u
                    Controls at f(t).
* \param[in] p
                    Parameters f^{0}.
* \param[out] gg
                    Nonlinear constaints as a
*
                    vector.
*
*/
virtual void NLCSTR(...) const;
/**
* \brief BOUND:
         Boundary constaints.
```

```
* \param[in]
             t0
                    Start timepoint f$t_0\f.
* \param[in] tF
                    End timepoint f_t_f.
* \param[in]
                    States at fy(t_f)f.
             уF
* \param[in]
                    States at f$y(t_0) f.
             y0
* \param[in]
                    Parameters f^p.
             р
* \param[out] psi
                    Boundary constaints as a
                    vector.
*
*/
virtual void BOUND(...) const;
/**
* \brief VARPHI:
        Target or objective function.
* \param[in] t0
                    Start timepoint f$t_0\f.
* \param[in] tF
                    End timepoint f_t_f.
                    States at f_y(t_f) f.
* \param[in] yF
* \param[in] y0
                    States at f$y(t_0) f.
* \param[in]
                    Parameters f^p.
             р
* \param[out] phi
                    Target or objective function
                     (vector of size 1).
*/
virtual void VARPHI(...) const;
virtual void D_ODE_X(...);
virtual void D_ODE_U(...);
virtual void D_ODE_P(...);
virtual void D_NLCSTR_X(...);
virtual void D_NLCSTR_U(...);
virtual void D_NLCSTR_P(...);
```

```
virtual void D_BOUND_XF(...);
virtual void D_BOUND_XO(...);
virtual void D_BOUND_P(...);
virtual void D_VARPHI_XF(...);
virtual void D_VARPHI_XO(...);
virtual void D_VARPHI_P(...);
virtual void L_DD_ODE_XX(...);
virtual void L_DD_ODE_UU(...);
virtual void L_DD_ODE_PP(...);
virtual void L_DD_ODE_XU(...);
virtual void L_DD_ODE_XP(...);
virtual void L_DD_ODE_UP(...);
virtual void L_DD_NLCSTR_XX(...);
virtual void L_DD_NLCSTR_UU(...);
virtual void L_DD_NLCSTR_PP(...);
virtual void L_DD_NLCSTR_XU(...);
virtual void L_DD_NLCSTR_XP(...);
virtual void L_DD_NLCSTR_UP(...);
virtual void L_DD_BOUND_XFXF(...);
```

```
virtual void L_DD_BOUND_XOXO(...);
virtual void L_DD_BOUND_PP(...);
virtual void L_DD_BOUND_XFXO(...);
virtual void L_DD_BOUND_XFP(...);
virtual void L_DD_BOUND_XOP(...);
virtual void L_DD_VARPHI_XFXF(...);
virtual void L_DD_VARPHI_XFXF(...);
virtual void L_DD_VARPHI_XOXO(...);
virtual void L_DD_VARPHI_PP(...);
virtual void L_DD_VARPHI_PP(...);
virtual void L_DD_VARPHI_XFXO(...);
virtual void L_DD_VARPHI_XFP(...);
virtual void L_DD_VARPHI_XFP(...);
```

Before calling OCPBASIC for the Minimum Energy Problem you may create a startsolution by integrating the Minimum Energy Problem. The Optimizer is created by the variable set:

- ME: Problem class for the Minimum Energy Problem.
- NGrid: Number of Gridpoints.
- t0: Starttime.
- tF: Endtime.
- LINSOL_LAPACK_BANDED_KKT: Used linear algebra solver.

The Optimizer can be called with a startsolution by the optimize routine.

Comment 4.1 The parameters of OCPBASIC from section 3.2 can be manipulated by getting the parameter class of the optimizer object.

4 EXAMPLES

```
#include "minimumenergyproblem.h"
#include <IPbasic/Errors/mycolordef.h>
#include <OCPbasic/Discretization/trapez.h>
#include <OCPbasic/OCP_Params.h>
#include <OCPbasic/Optimizer/InteriorPointMethod.h>
#include <OCPbasic/solution.h>
#include <iostream>
#include <OCPbasic/linalg/collectormatrix.h>
#include <OCPbasic/linalg/sparsematrix.h>
using IPbasic::TermColor;
int main()
{
int NGrid = 200000;
double t0 = 0.0;
double tF = 1.0;
std::cout << "Construct_Minimum_Energy_Problem..."</pre>
          << std::endl;
MinimumEnergyProblem ME;
std::cout << "Minimum_Energy_Problem_constructed!"</pre>
          << std::endl;
// Startsolution:
double u0[1];
u0[0] = -2.0;
double y0[3];
for (int k = 0; k < 3; ++k)
y0[k] = 0.0;
y0[1] = 1.0;
double h = (tF - t0) / ((double)NGrid);
OCPbasic::Solution<double> Sol(3, 1, 0, NGrid + 1, NGrid + 1);
Sol.integrateStartSolutionEuler(&ME, h, y0, u0);
OCPbasic::InteriorPointMethod<double> Opt(
    &ME, NGrid, t0, tF, OCPbasic::LINSOL_LAPACK_BANDED_KKT
    );
OCPbasic::OCP_Params<double> * Param =
```

```
Opt.getOCPParameter();
Param->mu_0
                         = 1e-3;
Param->MyRegularization = OCPbasic::REGULARIZE_NONE;
Param->PrintLev
                         = IPbasic::PRINT_MARKED;
Param->max_iter
                         = 1000;
if (Opt.optimize(Sol))
{
TermColor::set_COLOR_GREEN();
std::cout << "Success!" << std::endl;</pre>
TermColor::set_COLOR_DEFAULT();
}
else
{
TermColor::set_COLOR_RED();
std::cout << "Not_successfull!" << std::endl;</pre>
TermColor::set_COLOR_DEFAULT();
}
Sol.PrintFile_X("States.txt");
Sol.PrintFile_U("Controlls.txt");
return 0;
}
```

References

- A. Huber, M. Gerdts, and E. Bertolazzi. Structure exploitation in an interior-point method for fully discretized, state constrained optimal control problems. *Vietnam Journal of Mathematics*, Oct 2018.
- [2] A. K.-P. Huber. Methoden zur Berechnung optimaler Rennlinien im dynamischen Grenzbereich. PhD thesis, Universität der Bundeswehr München, 2020.